Fiscal Consolidation Policies and the Underground Economy: The Case of Greece

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Abstract

We revisit the effects of fiscal consolidation policies using a New Keynesian model with involuntary unemployment and an underground sector. We find that spending cuts induce a reallocation of production towards the formal sector, thus reducing tax evasion. On the other hand, tax hikes increase the incentives to produce in the less productive shadow sector, implying higher output and unemployment losses. We use the model to assess the recent fiscal consolidation plans in Greece. Our results corroborate the evidence of increasing levels of tax evasion during these consolidations and point to significant output and welfare losses, which could be reduced substantially by combating the underground economy.

JEL classification: H3, E6
Keywords: DSGE model, matching frictions, tax evasion, fiscal consolidation, policy analysis.

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1 Introduction

The recent fiscal crisis has sparked a considerable amount of research measuring the macroeconomic effects of fiscal consolidations.[1] This literature, however, has left aside a crucial political economy aspect, namely the presence of tax evasion. This is surprising, given that it is an important feature in many of the countries adopting consolidation policies, as Figure 1 shows. In addition, there is growing evidence that tax evasion has increased in recent years. For example, a recent report by the technical staff of the Spanish Finance Ministry (Gestha, 2014) indicates that the shadow economy increased by 6.8 percentage points between 2008 and 2012, reaching 24.6% of GDP. Using a model calibrated to firm-level data for Greece, Pappadà and Zylberberg (2014) show that the increase in tax evasion can explain three quarters of the revenue leakages following the 2010 VAT hikes, when only half of the expected increase in revenue was realized. Colombo et al. (2014) also provide empirical evidence of a rise in the underground economy in recent years by focusing on the role of the banking crisis. The aim of this paper is to revisit the effects of government expenditure cuts and labor tax hikes on output, unemployment and welfare, when tax evasion is present.

We treat tax evasion as synonymous with the shadow economy, which comprises *"all market-based, lawful production or trade of goods and services deliberately concealed from public authorities in order to evade either payment of income, value added or other taxes, or social security contributions"* (Buehn and Schneider, 2012, p.175-176).[2] Fiscal policy has an impact on the size of the underground economy, since it affects the incentives to tax evade both directly, through the tax burden, and indirectly, through its effects on the formal economy. Thus, a fiscal consolidation can have important secondary effects if it generates a reallocation of resources between the formal and informal sectors.

Many authors have studied whether it is preferable to rely on spending cuts or tax hikes when consolidating deficit. Overall, the findings are not conclusive. Using multi-year fiscal consolidation data for 17 OECD countries over the period 1980-2005, Alesina et al. (2013) show that expenditure-based adjustments are typically associated with mild and short-lived recessions, and in some cases with no recession at all, while tax-based corrections are followed by deep and prolonged recessions. On the other hand, Erceg and Lindé (2013) reach a different conclusion. Using a two-country Dynamic Stochastic General Equilibrium (DSGE) model of a currency union, they show that, in the short run, a spending cut depresses output by more than a labor tax hike, because of the limited accommodation by the central bank and the fixed exchange rate. However, this is reversed in the long run as real interest and exchange rates adjust towards their flexible price levels.

We reassess the effects of fiscal consolidations in a model with price stickiness, search and matching frictions, endogenous labor force participation, and tax evasion. The economy features a regular and an informal sector, and the transactions in the latter sector are not recorded by the government. Firms can hire informal labor to hide part of their production and evade payroll taxes.

[1] The implementation of the Maastricht Treaty in the mid 1990s initiated a wave of research on the effects of consolidations. For examples, see the survey in Perotti (1996).

[2] We use the terms "underground economy" and "shadow economy" interchangeably throughout the paper.

Households may also evade personal income taxation by reallocating their labor to the informal sector. In each period, there is a positive probability that irregular employment is detected, in which case the worker loses the job and the firm pays a fine. Following Erceg and Lindé (2013), either labor tax rates or government consumption expenditures react to the deviation of the debt-to-GDP ratio from a target value. Fiscal consolidation occurs when this target is hit by a negative shock.

We calibrate the model for Greece. Our findings indicate that the presence of tax evasion amplifies the negative effects of labor tax hikes on output and unemployment, while it mitigates those of expenditure cuts. Tax evasion implies that a larger increase in the tax rate is needed to reduce debt, and this amplifies the distortionary effects of the consolidation. Tax evasion further increases the output losses after a tax hike because workers and firms reallocate resources to the informal sector, increasing inefficiencies since this sector is less productive. On the other hand, government spending cuts reduce tax evasion. The spending cut creates a positive wealth effect, which increases consumption and investment and reduces labor force participation. This wealth effect leads to an investment boom, and a subsequent rise in the capital stock, and agents reallocate their labor search towards the formal sector, first, because it is more productive and, second, because the formal labor market has a higher matching efficiency and a lower job destruction rate. Hence, the share of underground employment in total employment is reduced. Relative to standard models, tax evasion increases the size of this wealth effect, thereby increasing the crowding-in of private consumption, and reducing output losses. Labor tax hikes are costly in terms of welfare, but spending cuts typically involve welfare gains, since private consumption increases and labor supply decreases. The latter result is reversed, however, if government spending directly enters the utility of households, or if agents are liquidity constrained.

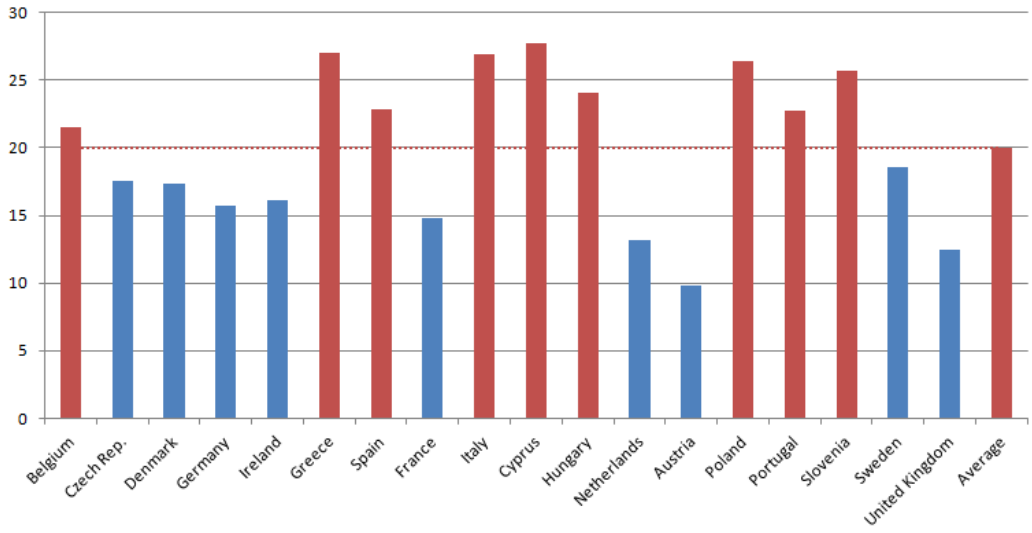
We use our model to evaluate the impact of the recent consolidation policies in Greece. Despite the fact that the consolidation plans rely heavily on spending cuts, the model predicts increasing levels of tax evasion, as well as prolonged recessions and substantial welfare losses. There have been considerable discussions in the policy arena about combating tax evasion. For example, members of the European Parliament organized an event focusing on tax evasion in Ljubljana in May 2013. The issue of reducing tax evasion also dominated the 2013 meeting of G8 leaders. To quantitatively evaluate the welfare impact of fighting tax evasion, we perform a counterfactual analysis of the consolidation plans when we increase the probability of tax audits. We find that the battle is worth fighting as it significantly reduces the welfare losses from fiscal consolidation.

The remainder of the paper is organized as follows. In the next section we develop the model and discuss the main theoretical results. Section 3 presents the policy evaluation exercise and Section 4 concludes.

2 The Model

We construct a DSGE model featuring search and matching frictions, endogenous labor decisions, and sticky prices in the short run. There are two types of firms in the economy: (i) competitive firms that produce intermediate goods in either the formal or informal sector, and (ii) monopolistic

Figure 1: Shadow Economy Estimates in European Countries



Shadow Economy (% GDP), Average over 1999-2010
Source: Schneider and Buehn (2012).

Note: The dotted line indicates the average for the countries considered.

retailers that use all intermediate varieties to produce differentiated retail goods, which are then costlessly aggregated into a final consumption good. Price rigidities arise at the retail level, while labor market frictions occur in the production of intermediate goods. Intermediate firms can choose to produce in the informal sector in order to evade the payroll taxes paid on formal employment. In each period, they face a probability of being inspected by the fiscal authorities and convicted of tax evasion, in which case they pay a penalty, and the employment match is terminated. There is a representative household consisting of formal and informal employees, unemployed jobseekers and labor force non-participants. Jobseekers can choose to search in the informal sector in order to evade income taxes. The household rents out its private capital to the intermediate firms, and purchases the final consumption good. The government collects taxes from the regular sector and uses them to finance public expenditures and the provision of unemployment benefits.

2.1 Labor markets

We account for the imperfections and transaction costs in the labor market by assuming that jobs are created through a matching function. For $j = F, I$ denoting the formal and informal sectors, let v_t^j be the number of vacancies and u_t^j the number of jobseekers in each sector. We assume matching functions of the form:

$$m_t^j = \mu_1^j (v_t^j)^{\mu_2} (u_t^j)^{1-\mu_2} \tag{1}$$

where we allow for differences in the efficiency of the matching process, μ_1^j, in the two sectors. In each sector we can define the probability of a jobseeker being hired, ψ_t^{hj}, and of a vacancy being filled, ψ_t^{fj}, as follows:

$$\psi_t^{hj} \equiv \frac{m_t^j}{u_t^j}, \qquad \psi_t^{fj} \equiv \frac{m_t^j}{v_t^j}$$

In each period, jobs in the formal sector are destroyed at a constant fraction, σ^F, and m_t^F new matches are formed. The law of motion of formal employment, n_t^F, is thus given by:

$$n_{t+1}^F = (1 - \sigma^F) n_t^F + m_t^F \tag{2}$$

In the informal sector there is an exogenous fraction of jobs destroyed in each period, σ^I, as well as a probability, ρ, that an informal employee loses their job due to an audit. The law of motion of informal employment, n_t^I, is given by:

$$n_{t+1}^I = (1 - \rho - \sigma^I) n_t^I + m_t^I \tag{3}$$

2.2 Households

The representative household consists of a continuum of infinitely lived agents. The members of the household derive utility from leisure, which corresponds to the fraction of members that are out of the labor force, l_t, and a consumption bundle, cc_t, defined as:

$$cc_t = [\alpha_1 (c_t)^{\alpha_2} + (1 - \alpha_1)(g_t)^{\alpha_2}]^{\frac{1}{\alpha_2}}$$

where g_t denotes public consumption, taken as exogenous by the household, and c_t is private consumption. The elasticity of substitution between the private and public goods is given by $\frac{1}{1-\alpha_2}$.[3] The instantaneous utility function is given by:

$$U(cc_t, l_t) = \frac{cc_t^{1-\eta}}{1-\eta} + \Phi \frac{l_t^{1-\varphi}}{1-\varphi}$$

where η is the inverse of the intertemporal elasticity of substitution, $\Phi > 0$ is the relative preference for leisure, and φ is the inverse of the Frisch elasticity of labor supply.

At any point in time, a fraction n_t^F (n_t^I) of the household members are formal (informal) employees. Campolmi and Gnocchi (2014), Brückner and Pappa (2012) and Bermperoglou et al. (2014) have added a labor force participation choice in New Keynesian models of equilibrium unemployment. Following Ravn (2008), the participation choice is modelled as a trade-off between the cost of giving up leisure and the prospect of finding a job. In particular, the household chooses the fraction of the unemployed actively searching for a job, u_t, and the fraction which are out of

[3] When α_2 approaches one, c_t and g_t are perfect substitutes. They are instead perfect complements if α_2 tends to minus infinity. $\alpha_2 = 0$ nests the Cobb-Douglas specification.

the labor force and enjoying leisure, l_t, so that:

$$n_t^F + n_t^I + u_t + l_t = 1 \tag{4}$$

The household chooses the fraction of jobseekers searching in each sector: a share s_t of jobseekers look for a job in the informal sector, while the remainder, $(1 - s_t)$, seek employment in the formal sector. That is, $u_t^I \equiv s_t u_t$ and $u_t^F \equiv (1 - s_t)u_t$.

The household owns the capital stock, which evolves over time according to:

$$k_{t+1} = i_t + (1 - \delta)k_t - \frac{\omega}{2}\left(\frac{k_{t+1}}{k_t} - 1\right)^2 k_t \tag{5}$$

where i_t is investment, δ is a constant depreciation rate and $\frac{\omega}{2}\left(\frac{k_{t+1}}{k_t} - 1\right)^2 k_t$ are adjustment costs.

The intertemporal budget constraint is given by:

$$(1 + \tau_t^c)c_t + i_t + \frac{B_{t+1}\pi_{t+1}}{R_t} \leq r_t k_t + (1 - \tau_t^n)w_t^F n_t^F + w_t^I n_t^I + \varpi u_t^F + B_t + \Pi_t^p - T_t \tag{6}$$

where $\pi_t \equiv p_t/p_{t-1}$ is the gross inflation rate, w_t^j, $j = F, I$, are the real wages in the two sectors, r_t is the real return on capital, ϖ denotes unemployment benefits, available only to formal jobseekers (see e.g. Boeri and Garibaldi, 2007), B_t is the real government bond holdings, R_t is the gross nominal interest rate, Π_t^p are the profits of the monopolistic retailers, discussed below, and τ_t^c, τ_t^n and T_t represent taxes on private consumption, labor income and lump-sum taxes respectively.

The household maximizes expected lifetime utility subject to (1) for each j, (2), (3), (4), (5), and (6). Taking as given n_t^j, they choose u_t, s_t (which together determine l_t) and n_{t+1}^j, as well as c_t, k_{t+1} and B_{t+1}.

It is convenient to define the marginal value to the household of having an additional member employed in each sector, as follows:

$$V_{n^F t}^h = \lambda_{ct} w_t^F (1 - \tau_t^n) - \Phi l_t^{-\varphi} + (1 - \sigma^F)\lambda_{n^F t} \tag{7}$$

$$V_{n^I t}^h = \lambda_{ct} w_t^I - \Phi l_t^{-\varphi} + (1 - \rho - \sigma^I)\lambda_{n^I t} \tag{8}$$

where $\lambda_{n^F t}$, $\lambda_{n^I t}$ and λ_{ct} are the multipliers in front of (2), (3) and (6) respectively.

2.3 Production

2.3.1 Intermediate goods firms

Intermediate goods are produced with two different technologies:

$$x_t^F = (A_t^F n_t^F)^{1-\alpha^F}(k_t)^{\alpha^F} \tag{9}$$

$$x_t^I = (A_t^I n_t^I)^{1-\alpha^I} \tag{10}$$

where A_t^j denotes total factor productivity in sector j. Following the literature, we assume that the informal production technology uses labor inputs only (see e.g. Busato and Chiarini, 2004).

Firms maximize the discounted value of future profits, subject to (2) and (3). That is, they take the number of workers currently employed in each sector, n_t^j, as given and choose the number of vacancies posted in each sector, v_t^j, so as to employ the desired number of workers next period, n_{t+1}^j. Here, firms adjust employment by varying the number of workers (extensive margin) rather than the number of hours per worker (intensive margin). According to Hansen (1985), most of the employment fluctuations arise from movements in this margin. Firms also decide the amount of private capital, k_t, needed for production. They face a probability, ρ, of being inspected by the fiscal authorities, convicted of tax evasion and forced to pay a penalty, which is a fraction, γ, of their total revenues. We assume that, once they are produced, there is no differentiation between intermediate goods from the different sectors. In other words, we assume that formal and informal goods are perfect substitutes, so that they are sold at the same price, p_t^x (see e.g. Orsi et al., 2014). Hence the problem of an intermediate firm is summarized by the following Bellman equation:

$$Q(n_t^F, n_t^I) = \max_{k_t, v_t^F, v_t^I} \Big\{ (1-\rho\gamma)\, p_t^x (x_t^F + x_t^I) - (1+\tau_t^s) w_t^F n_t^F - w_t^I n_t^I - r_t k_t \\ - \kappa^F v_t^F - \kappa^I v_t^I + E_t \left[\Lambda_{t,t+1} Q(n_{t+1}^F, n_{t+1}^I) \right] \Big\}$$

where τ_t^s is a payroll tax, κ^j is the cost of posting a new vacancy in sector j, and $\Lambda_{t,t+1} \equiv \beta \frac{U_{cc,t+1}}{U_{cc,t}} = \beta \left(\frac{cc_{t+1}}{cc_t} \right)^{-\eta}$ is a discount factor. The first-order conditions are:

$$r_t = (1-\rho\gamma)\, p_t^x \left(\frac{\alpha^F x_t^F}{k_t} \right) \tag{11}$$

$$\frac{\kappa^F}{\psi_t^{fF}} = E_t \Lambda_{t,t+1} \left[(1-\rho\gamma)\, p_{t+1}^x (1-\alpha^F) \frac{x_{t+1}^F}{n_{t+1}^F} - (1+\tau_{t+1}^s) w_{t+1}^F + \frac{(1-\sigma^F)\kappa^F}{\psi_{t+1}^{fF}} \right] \tag{12}$$

$$\frac{\kappa^I}{\psi_t^{fI}} = E_t \Lambda_{t,t+1} \left[(1-\rho\gamma)\, p_{t+1}^x (1-\alpha^I) \frac{x_{t+1}^I}{n_{t+1}^I} - w_{t+1}^I + \frac{(1-\rho-\sigma^I)\kappa^I}{\psi_{t+1}^{fI}} \right] \tag{13}$$

According to (11)-(13), the net value of the marginal product of private capital should equal the real rental rate and the expected marginal cost of hiring a worker in each sector j should equal the expected marginal benefit. The latter includes the net value of the marginal product of labor minus the wage, augmented by the payroll tax in the formal sector, plus the continuation value.

For convenience, we define the value of the marginal formal and informal job for the intermediate firm:

$$V_{n^F t}^f = (1-\rho\gamma)\, p_t^x (1-\alpha^F) \frac{x_t^F}{n_t^F} - (1+\tau_t^s) w_t^F + \frac{(1-\sigma^F)\kappa^F}{\psi_t^{fF}} \tag{14}$$

$$V_{n^I t}^f = (1-\rho\gamma)\, p_t^x (1-\alpha^I) \frac{x_t^I}{n_t^I} - w_t^I + \frac{(1-\rho-\sigma^I)\kappa^I}{\psi_t^{fI}} \tag{15}$$

2.3.2 Retailers

There is a continuum of monopolistically competitive retailers indexed by i on the unit interval. Retailers buy intermediate goods and differentiate them with a technology that transforms one unit of intermediate goods into one unit of retail goods, and thus the relative price of intermediate goods, p_t^x, coincides with the real marginal cost faced by the retailers. Let y_{it} be the quantity of output sold by retailer i. The final consumption good can be expressed as:

$$y_t = \left[\int_0^1 (y_{it})^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} \tag{16}$$

where $\epsilon > 1$ is the constant elasticity of demand for retail goods. The final good is sold at a price $p_t = \left[\int_0^1 p_{it}^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$. The demand for each intermediate good depends on its relative price and on aggregate demand:

$$y_{it} = \left(\frac{p_{it}}{p_t}\right)^{-\epsilon} y_t \tag{17}$$

Following Calvo (1983), we assume that in any given period each retailer can reset its price with a fixed probability $(1-\chi)$. Hence, the price index is given by:

$$p_t = \left[(1-\chi)(p_t^*)^{1-\epsilon} + \chi(p_{t-1})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{18}$$

Firms that are able to reset their price choose p_{it}^* so as to maximize expected profits given by:

$$E_t \sum_{s=0}^{\infty} \chi^s \Lambda_{t,t+s}(p_{it}^* - p_{t+s}^x) y_{it+s}$$

The resulting expression for p_{it}^* is:

$$p_{it}^* = \frac{\epsilon}{\epsilon-1} \frac{E_t \sum_{s=0}^{\infty} \chi^s \Lambda_{t,t+s} p_{t+s}^x y_{it+s}}{E_t \sum_{s=0}^{\infty} \chi^s \Lambda_{t,t+s} y_{it+s}} \tag{19}$$

2.4 Government

Government expenditure consists of consumption purchases and unemployment benefits, while revenues come from the collected fines and the payroll, consumption, and labor income taxes, as well as the lump-sum taxes. The government deficit is therefore defined by:

$$DF_t = g_t + \varpi u_t^F - TR_t - \rho\gamma p_t^x (x_t^F + x_t^I) \tag{20}$$

where $TR_t \equiv (\tau_t^n + \tau_t^s) w_t^F n_t^F + \tau_t^c c_t + T_t$ denotes tax revenues.

The government budget constraint is given by:

$$B_t + DF_t = R_t^{-1} B_{t+1} \pi_{t+1} \tag{21}$$

We assume that T_t, τ_t^s, and τ_t^c are constant and fixed at their steady state levels, and we do not consider them as active instruments for fiscal consolidation. In our model, the effects of payroll taxes are very similar to labor income taxes. Consumption taxes can have different effects, but they generally constitute a relatively small source of tax revenues. Thus, in line with Erceg and Lindé (2013), the government has two potential fiscal instruments, g and τ^n. We consider each instrument separately, assuming that if one is active, the other remains fixed at its steady state value. For $\Psi \in \{g, \tau^n\}$, we assume fiscal rules of the form:

$$\Psi_t = \Psi^{(1-\beta_{\Psi 0})} \Psi_{t-1}^{\beta_{\Psi 0}} \exp\{(1-\beta_{\Psi 0})[\beta_{\Psi 1}(b_t - b_t^*) + \beta_{\Psi 2}(\Delta b_{t+1} - \Delta b_{t+1}^*)]\} \tag{22}$$

where $b_t = \frac{B_t}{y_t}$ is the debt-to-GDP ratio, and b_t^* is the target value for this ratio, given by the AR(2) process:

$$\log b_{t+1}^* - \log b_t^* = \mu_b + \rho_1(\log b_t^* - \log b_{t-1}^*) - \rho_2 \log b_t^* - \varepsilon_t^b \tag{23}$$

where ε_t^b is a white noise shock representing a fiscal consolidation.

2.5 Closing the model

Monetary Policy There is an independent monetary authority that sets the nominal interest rate as a function of current inflation according to the rule:

$$R_t = R \exp\{\zeta_\pi(\pi_t - 1)\} \tag{24}$$

where R is the steady state value of the nominal interest rate.

Goods Markets Total output must equal private and public demand. The aggregate resource constraint is thus given by:

$$y_t = c_t + i_t + g_t + \kappa^F v_t^F + \kappa^I v_t^I \tag{25}$$

The aggregate price index, p_t, is given by (18) and (19). The return on private capital, r_t, adjusts so that the capital demanded by the intermediate goods firm, given by (11), is equal to the stock held by the household.

Bargaining over wages Wages in both sectors are determined by ex-post (after matching) Nash bargaining. Workers and firms split rents and the part of the surplus they receive depends on their bargaining power. We denote by $\vartheta^j \in (0,1)$ the firms' bargaining power in sector j. The Nash bargaining problem is to maximize the weighted sum of log surpluses:

$$\max_{w_t^j} \left\{ (1-\vartheta^j) \log V_{n^j t}^h + \vartheta^j \log V_{n^j t}^f \right\}$$

where $V^h_{n^j t}$ and $V^f_{n^j t}$ are defined in equations (7), (8), (14) and (15). As shown in the Appendix, wages are given by:

$$w_t^F = \frac{(1-\vartheta^F)}{(1+\tau_t^s)}\left((1-\rho\gamma)\,p_t^x(1-\alpha^F)\frac{x_t^F}{n_t^F} + \frac{(1-\sigma^F)\kappa^F}{\psi_t^{fF}}\right) + \frac{\vartheta^F}{\lambda_{ct}(1-\tau_t^n)}\left(\Phi l_t^{-\varphi} - (1-\sigma^F)\lambda_{n^F t}\right) \tag{26}$$

$$w_t^I = (1-\vartheta^I)\left((1-\rho\gamma)\,p_t^x(1-\alpha^I)\frac{x_t^I}{n_t^I} + \frac{(1-\rho-\sigma^I)\kappa^I}{\psi_t^{fI}}\right) + \frac{\vartheta^I}{\lambda_{ct}}\left(\Phi l_t^{-\varphi} - (1-\rho-\sigma^I)\lambda_{n^I t}\right) \tag{27}$$

2.6 Calibrating the Model

We calibrate the model using annual Greek data for the period 1982-2006.[4] Table 1 displays the values used. We calibrate the labor force participation and unemployment rate in the formal sector to match the observed average values. Thus, we set official labor force participation, $lf \equiv n^F + u^F$, equal to 64% and the official unemployment rate to 10%. We fix the separation rate $\sigma^F = 0.07$. We set the probability of filling a vacancy in the formal sector $\psi^{fF} = 0.96$, and the matching elasticity with respect to vacancies $\mu_2 = 0.7$, which is close to the estimate obtained in Peracchi and Viviano (2004).

The capital depreciation rate, δ, is set equal to 0.088. Following the literature, we set the discount factor $\beta = 0.96$. The elasticity of demand for retail goods, ϵ, is set such that the gross steady state markup, $\frac{\epsilon}{\epsilon-1}$, is equal to 1.25, and the price of the final good is normalized to one. The TFP parameter in the formal sector, A^F, is normalized to one, and the capital share $\alpha^F = 0.36$. We set the vacancy costs in the formal sector $\kappa^F = 0.14$, and the payroll tax rate $\tau^s = 16\%$, close to the value used in Orsi et al. (2014).

In the informal sector, we assume that TFP is lower than the formal sector by setting $A^I = 0.6$. According to Restrepo-Echavarria (2014), the fact that the informal sector has restricted access to credit leads to fewer resources being devoted to research and development, or to absorbing technology spillovers, which in turn reduces productivity. Also, both Boeri and Garibaldi (2007) and Orsi et al. (2014) emphasize empirical evidence suggesting that the workers in the informal sector have lower education levels.[5]

We set the share of informal employment to total employment equal to 0.14, and we set $\alpha^I = 0.4$, implying the share of shadow output to total output $\frac{y^I}{y} = 16\%$. We set the exogenous job destruction rate in the informal sector $\sigma^I = 0.0545$, the probability of filling a vacancy in the informal sector $\psi^{fI} = 0.05$, and the vacancy cost in the informal sector $\kappa^I = 0.13$. These values yield a relatively small wage premium for the formal sector, $\frac{w^I}{w^F} = 0.98$, in line with the literature. The probability of audit and the fraction of total revenues paid as a fine in the event of an audit are set as follows: $\rho = 0.02$, close to the value used in Boeri and Garibaldi (2007), and $\gamma = 0.3$.

We set the replacement rate $\frac{\varpi}{w^F} = 0.35$, close to the estimates in Martin (1996), and used by Fugazza and Jacques (2004). Government spending as a share of GDP and the remaining tax

[4] Details of the calibration exercise are in the Appendix.

[5] Orsi et al. (2014) also note the equivalence of assuming lower productivity in the informal sector to assuming a cost for concealing production.

Table 1: Calibration Values

Parameter	Description	Full Model
β	Discount Factor	0.96
δ	Depreciation Rate	0.088
α_1	Share of Private Consumption in Utility	1
η	Inverse Elasticity of Intertemporal Substitution	2
φ	Inverse Frisch Elasticity of Labor Supply	2
Φ	Relative Utility from Leisure	0.7
lf	Official Labor Force Participation	0.64
$\frac{u^F}{lf}$	Official Unemployment Rate	0.1
$\frac{u}{1-l}$	Actual Unemployment Rate	0.09
s	Share of Informal Jobseekers to Total	0.10
$\frac{n^I}{n}$	Share of Informal Employment to Total	0.14
σ^F	Exogenous Job Destruction Rate - Formal Sector	0.07
σ^I	Exogenous Job Destruction Rate - Informal Sector	0.0545
ρ	Auditing Probability	0.02
μ_1^F	Matching Efficiency - Formal Sector	0.85
μ_1^I	Matching Efficiency - Informal Sector	0.12
μ_2	Elasticity of Matching to Vacancies	0.7
ψ^{fF}	Probability of Filling a Vacancy - Formal Sector	0.96
ψ^{fI}	Probability of Filling a Vacancy - Informal Sector	0.05
ψ^{hF}	Probability of Finding a Job - Formal Sector	0.63
ψ^{hI}	Probability of Finding a Job - Informal Sector	0.91
A^F	TFP - Formal Sector	1
A^I	TFP - Informal Sector	0.6
α^F	Capital Share - Formal Sector	0.36
α^I	Production Function Parameter - Informal Sector	0.4
$\frac{y^I}{y}$	Share of Underground Output in Total	0.16
κ^F	Vacancy Costs - Formal Sector	0.14
$\frac{\kappa^F}{w^F}$	Vacancy Costs/Wage	0.21
κ^I	Vacancy Costs - Informal Sector	0.13
ϵ	Price Elasticity of Demand	5
ϑ^F	Firm's Bargaining Power - Formal Sector	0.22
ϑ^I	Firm's Bargaining Power - Informal Sector	0.80
$\frac{w^I}{w^F}$	Formal/Informal Wage Differentials	0.98
$\frac{g}{y}$	Government Expenditure-to-GDP Ratio	0.05
$\frac{\varpi}{w^F}$	Replacement Rate	0.35
τ^n	Labor Income Tax Rate	0.4
τ^s	Payroll Tax Rate	0.16
τ^c	Consumption Tax Rate	0.18
γ	Proportional Fine in Case of Auditing	0.3
$\frac{DF}{y}$	Deficit-to-GDP Ratio	-0.04
b	Debt-to-GDP Ratio	1.45
ρ_1, ρ_2	Debt-to-GDP Target Parameters	0.85, 0.0001
χ	Price Stickiness	0.25
ω	Capital Adjustment Costs	0.5
ζ_π	Taylor Rule Parameter	1.5

rates are set as follows: $\frac{g}{y} = 5\%$, $\tau^n = 40\%$, in line with Orsi et al. (2014), and $\tau^c = 18\%$. The steady state debt-to-GDP ratio $b = 145\%$.

We begin by assuming purely wasteful government expenditure, setting $\alpha_1 = 1$, and will consider utility enhancing government spending as an extension. Regarding the inverse elasticity of intertemporal substitution, η, much of the literature cites the econometric estimates of Hansen and Singleton (1983), which place it "between 0 and 2", and often choose a value greater than unity. In our calibration, we set $\eta = 2$ and we perform sensitivity analysis by considering η equal to 0.5 and 1. The inverse of the Frisch elasticity, φ, is set equal to 2. Finally, we set the inflation targeting parameter in the Taylor rule $\zeta_\pi = 1.5$, the capital adjustment costs $\omega = 0.5$ and the price-stickiness parameter $\chi = 0.25$.

2.7 Results

We present responses following a negative debt-target shock (see Erceg and Lindé, 2013). We compare the effects of a 5% reduction in the desired long run debt target, which is achieved after 10 years, either through a fall in government consumption expenditure, or a hike in labor tax rates.[6]

2.7.1 Dynamics in a Model without Tax Evasion

As a benchmark, we begin by analyzing the responses of a standard model where tax evasion is absent, shown in Figure 2.

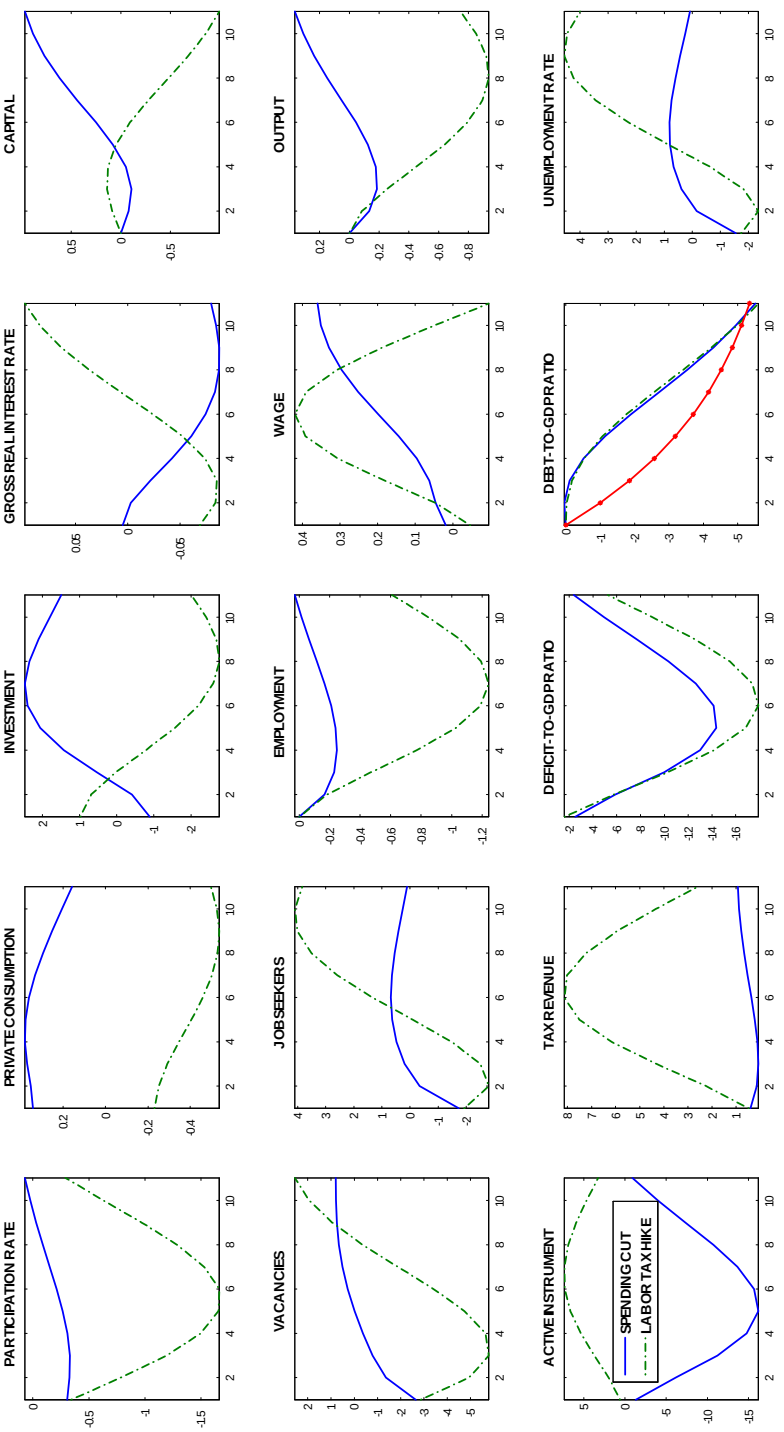
A consolidation carried out through a fall in government spending has two effects. Firstly, there is a negative demand effect for firms, which leads, in the presence of nominal rigidities, to a fall in labor demand and hence in vacancies. Second, there is a positive wealth effect for the household, which increases consumption and investment and reduces labor force participation. Given the drop in both labor demand and supply, employment falls and the wage rate rises. Output falls in the short run, but increases in the medium and long run because investment, and hence the capital stock, increases. The unemployment rate reflects movements in the number of jobseekers: it falls on impact, but then increases as employment and wages adjust.

When the fiscal consolidation is carried out through a labor tax hike, there is a negative wealth effect for the household which makes consumption fall, and investment fall with a lag. However, as the return from employment falls, there is a substitution effect which outweighs the wealth effect, and leads to a decrease in labor force participation. The fall in private demand induces firms to contract their labor demand, again expressed through a drop in vacancies. Employment and output fall, and the responses are significantly larger and more persistent than in the case of spending cuts, due to the fall in investment.

Thus, our benchmark model seems to be consistent with the empirical evidence of Alesina et al. (2013); spending cuts are accompanied by mild and short-lived recessions, while tax hikes lead to more prolonged and deep recessions.

[6] For comparison purposes, throughout this section we adjust the parameters of the policy rules for each case to ensure that the debt target is met after 10 years.

Figure 2: Impulse Response Functions of the Benchmark Model (without Tax Evasion)



2.7.2 Dynamics in a Model with Tax Evasion

We now move to the full model with tax evasion. Figure 3 presents the responses of the formal sector and of fiscal variables, and Figure 4 shows the responses in the informal sector.

To start with, notice that the response of the formal sector is qualitatively similar to the benchmark model. However, there is an additional channel at play. For the case of tax hikes, unemployed jobseekers reallocate their labor supply and the intermediate firms reallocate their labor demand towards the informal sector. Tax hikes provide direct incentives for jobseekers to search in the informal sector because of the higher tax rates in the formal sector. At the same time, intermediate firms find it profitable to post vacancies in the informal sector because of the fall in the informal wage. The fall in investment, and hence the capital stock, lowers the productivity differential between the two sectors, and further provides incentives for agents to reallocate to the informal sector. As a result, shadow employment as a share of total employment increases.

For the case of expenditure cuts, the negative demand effect of the spending cut affects both formal and informal production, leading to a reduction in labor demand in both sectors. Similarly, as labor force participation falls, there is a reduction in unemployed jobseekers in both sectors. This causes a contraction in total employment. Moreover, there is a reallocation of labor towards the formal sector; underground employment as a share of total employment falls. This happens for two reasons. Firstly, the formal labor market has a higher matching efficiency, and a lower job destruction rate. Secondly, in addition to having a higher TFP level, the rise in the capital stock further increases the productivity of the formal sector relative to the informal sector. In order to take advantage of these efficiency gains, and thus mitigate the negative effects of the fiscal contraction, agents optimally choose to reallocate towards the formal sector.

2.7.3 A More Detailed Comparison

Figure 5 compares the responses of output, the unemployment rate and welfare in the two models.[7] For spending cuts, shown in the top panel, the presence of tax evasion generates smaller losses in output, a drop in the unemployment rate at all horizons, and larger welfare gains. With tax evasion, the tax adjustments required to achieve a given change in deficit are larger, and thus, following a spending cut, taxes in the future are expected to fall by more. In other words, there is an amplification of the positive wealth effect. Hence the rise in consumption and the fall in labor force participation are larger relative to the model without tax evasion, making welfare gains larger. The increased crowding-in of private consumption mitigates the negative demand effect for the firms, thereby mitigating output losses. The larger reduction in labor force participation implies a fall in the number of formal jobseekers, and hence in the official unemployment rate, at all horizons.

For tax hikes, shown in the middle panel, the presence of tax evasion amplifies the output losses, particularly in the long run. This is due to the loss of tax revenue from tax evasion, implying that larger increases in tax rates are needed to reduce debt-to-GDP. This increases the distortionary effects of the consolidation, leading to a larger drop in labor force participation,

[7] Welfare is computed as per-period steady state consumption equivalents.

Figure 3: Impulse Response Functions of the Full Model (with Tax Evasion)

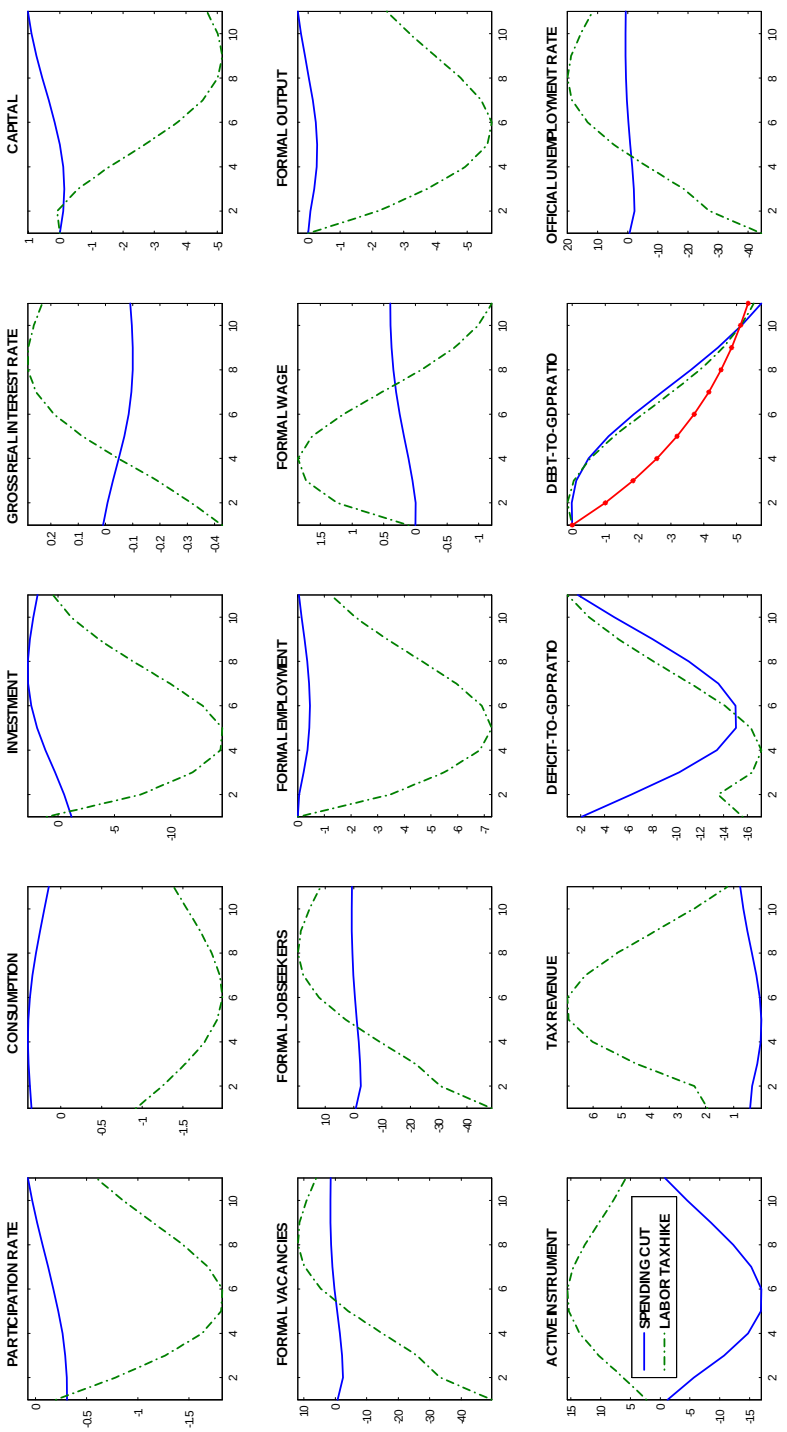


Figure 4: Impulse Response Functions of the Full Model - Underground Sector

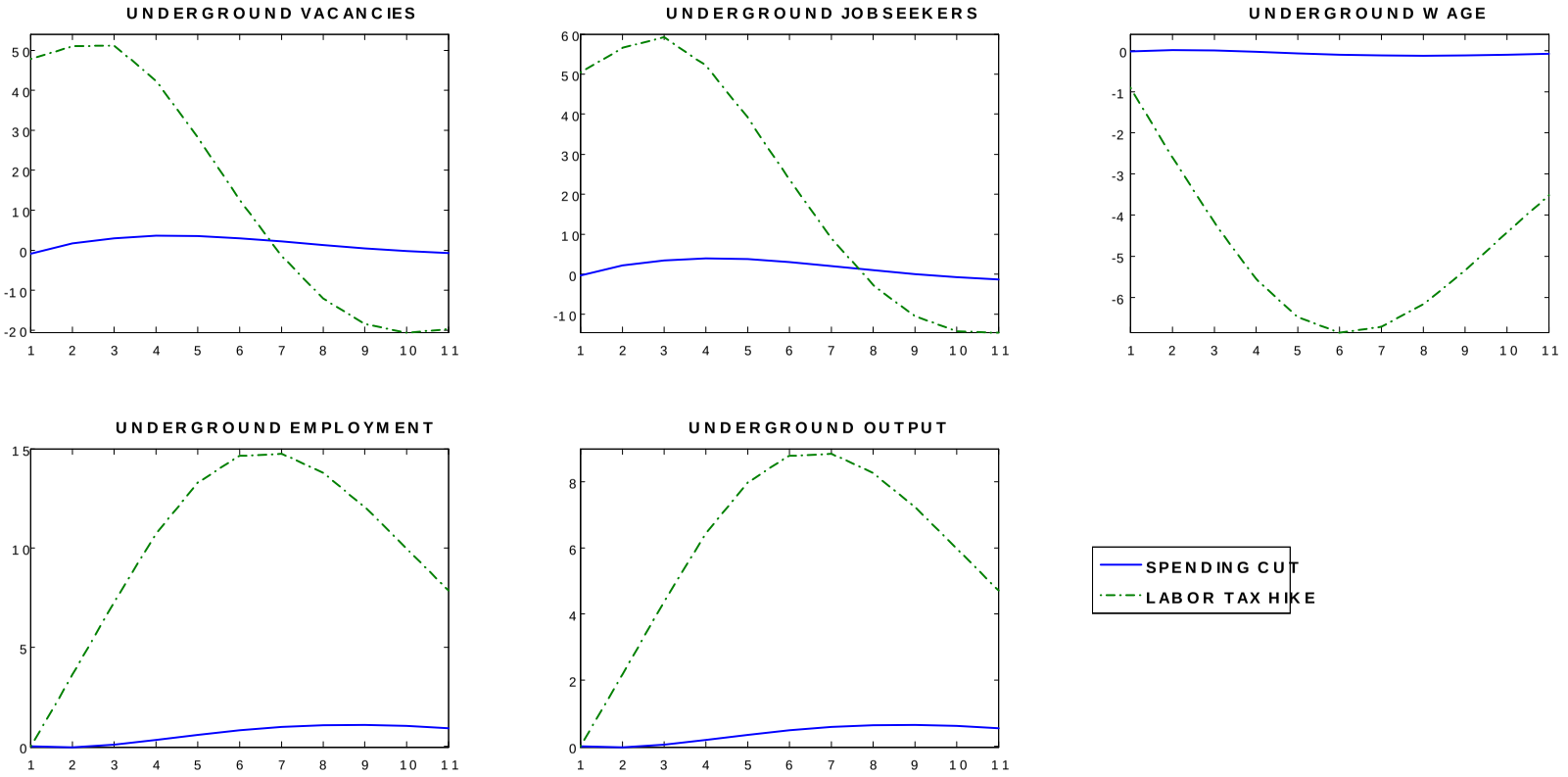
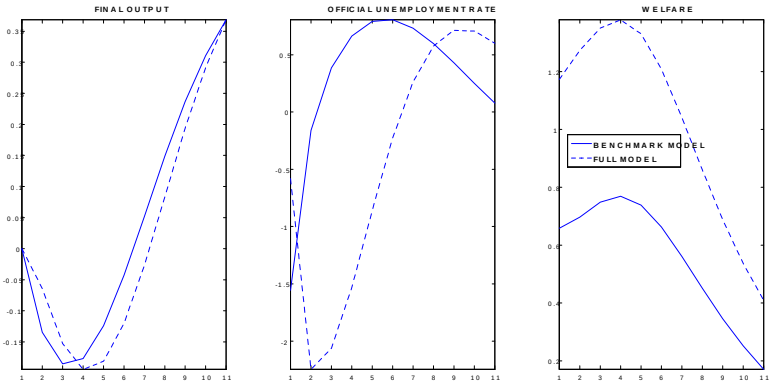
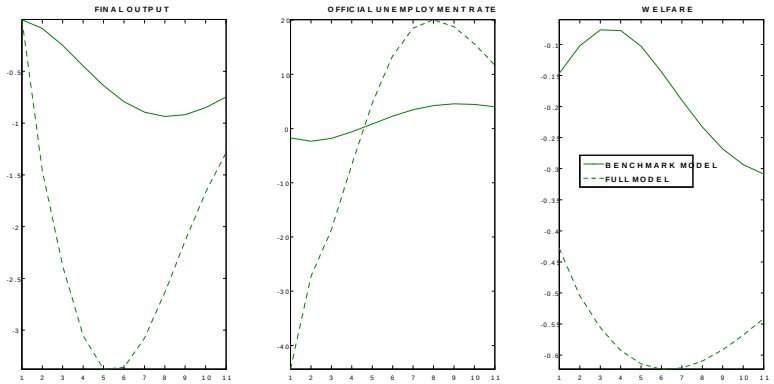


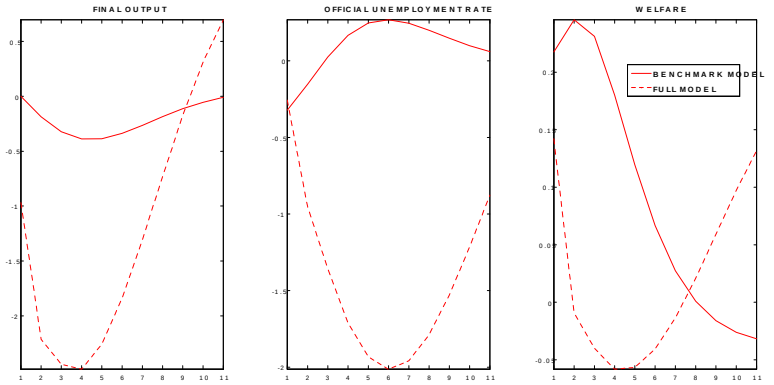
Figure 5: Comparison of Benchmark and Full Model



Government Expenditure Cuts



Labor Tax Hikes



Mixed Consolidation

private consumption and investment. In addition, the reallocation towards the informal sector increases the inefficiencies due to the lower productivity in this sector. Thus, there is a larger contraction in the formal sector, which is also evident in the response of the official unemployment rate: the initial fall is amplified as jobseekers drop out of the formal sector, and the rise in the long run is higher as firms post fewer vacancies in this sector. Furthermore, tax hikes lead to welfare losses. Initially, these losses are lower with tax evasion, but in the medium and long run, as consumption falls increasingly, we obtain higher losses.

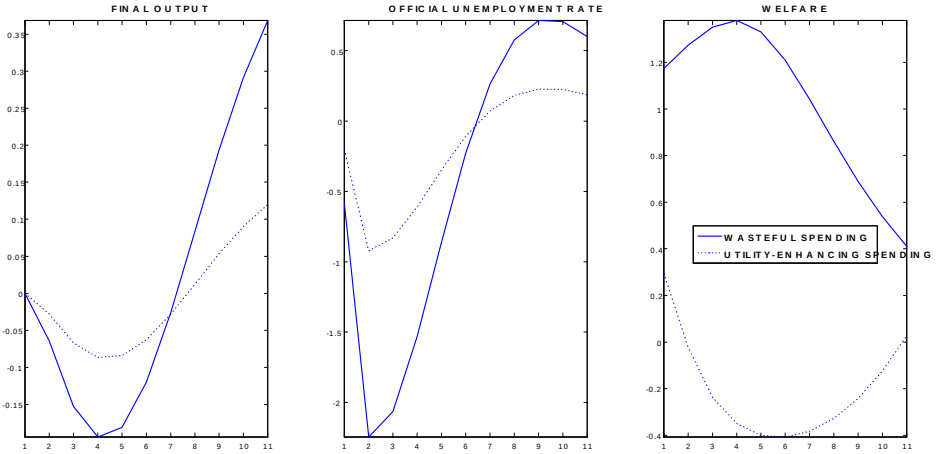
The bottom panel depicts the responses in the case of a mixed consolidation. Here, we allow both policy instruments, g and τ^n, to move simultaneously to reduce the deficit, which follows the debt-targeting rule. We fix the policy mix such that a fraction a of the reductions in deficit come from expenditure cuts and $(1-a)$ from revenue enhancements, and set $a = 0.5$. In this case, the responses of consumption and investment are determined by the competing positive and negative wealth effects from the two instruments, and the presence of tax evasion plays an important role in determining this relative strength. In the benchmark model, the positive wealth effect of the expenditure cut is dominant and consumption rises for several periods. When there is tax evasion, this is no longer true and consumption and investment fall in all periods. Hence, as in the case of tax hikes, output and unemployment responses are amplified in the presence of tax evasion Moreover, the welfare gains obtained from mixed consolidation packages in the benchmark model turn into welfare losses in the model with tax evasion.

2.7.4 Sensitivity Analysis

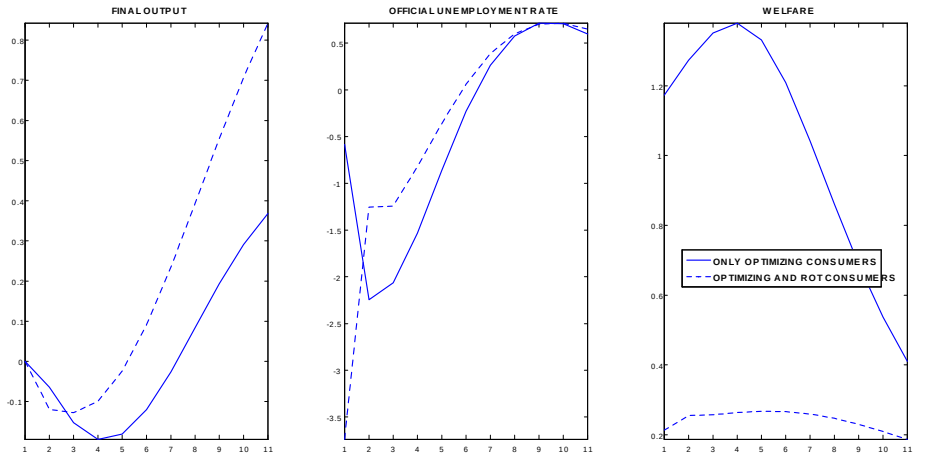
Spending cuts The effects of expenditure cuts depend crucially on some modeling assumptions. For instance, assuming that government expenditures provide a public good, which is consumed by households, can change the welfare implications of spending cuts. To illustrate this point, we set $\alpha_1 = 0.85$ and $\alpha_2 = -0.25$, so that private and public spending are weak complements. Figure 6 compares the results of this case with those obtained with wasteful government spending. In the case of utility-enhancing expenditures, a spending cut directly reduces the consumption bundle and households are forced to offset this fall by further increasing private consumption. Thus we see a larger crowding-in of private consumption, which mitigates the output and unemployment effects of spending cuts. However, the welfare effects are reversed: the drop in the consumption bundle causes welfare to fall for several periods.

The presence of liquidity constrained consumers has been shown to play an important role in determining the response of private consumption to a government spending cut (see Galí et al., 2007). To explore how the presence of liquidity constrained consumers can affect our model, we assume a fraction of rule of thumb (ROT) household members, which we set equal to 65%, in line with the Martin and Philippon (2014). As shown in the bottom panel of Figure 6, output and unemployment responses are amplified and welfare gains are mitigated following a spending cut. The presence of ROT agents reduces the positive wealth effect that the fiscal contraction generates, which implies a smaller increase in consumption and, hence, welfare, and a larger contraction in output.

Figure 6: Sensitivity Analysis for Spending Cuts in the Full Model



Utility-Enhancing Government Spending



Rule of Thumb (ROT) Consumers

Tax hikes A large body of the literature, initiated by Feldstein (1999), has argued that the costs of labor taxes can be summarized by the elasticity of taxable income with respect to the net of tax share. Given the complexity of our theoretical framework, calculating the magnitude of this elasticity can yield further insights about the effects of tax hikes in the presence of tax evasion and corruption. We compute the taxable income elasticity by dividing the cumulative response of taxable income by the cumulative response of the net tax share up to the point that the responses of tax rates return to zero in the two models. For the benchmark model, the elasticity of taxable income equals 0.2, while incorporating tax evasion and corruption in the analysis almost doubles this elasticity to 0.5.[8] This is not surprising, given that we are allowing workers to move out of taxable work, in the formal sector, not only by leaving the workforce but also by working in the informal sector. Data also suggests higher estimates of the taxable income elasticity in countries with more tax evasion and corruption. For example, Kleven and Schultz (2014) provide an estimate for Denmark equal to 0.09 for this elasticity and, using the same methodology, Arrazola et al. (2014) report a taxable income elasticity equal to 1.5 in Spain.

Of course many of our parameter choices affect the estimated value of the taxable income elasticity and so, in turn, our conclusions about the effects of tax hikes in a model with tax evasion and corruption. To investigate this, we first consider the inverse of the intertemporal elasticity of substitution, η. Smaller values of η imply higher values of the long run elasticity of taxable income. Accordingly, the higher values of the taxable income elasticity, implied by the lower η, are associated with higher output and welfare losses, as well as higher unemployment in the medium and long run, as seen in the top row of Figure 7.

Next, we use alternative values for the inverse of the Frisch elasticity of labor supply, φ. The value of the labor supply elastictity determines the size of the substitution effect and this, in turn, affects the taxable income elasticity in our model. Higher values of the labor supply elasticity, meaning lower values of φ, are associated with higher values of the taxable income elasticity. Results presented in the second row of Figure 7 indicate that output losses and medium and long run unemployment increase with the Frisch elasticity.[9]

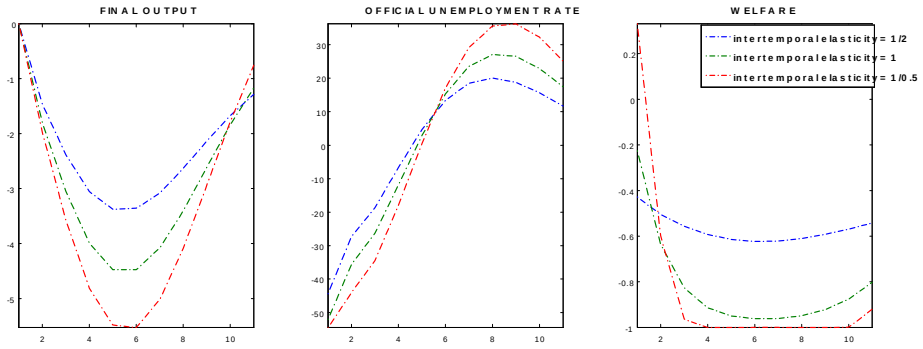
Finally, the incentives to tax evade are also affected by the probability of detection. We investigate the role of the detection probability in the last row of Figure 7. Intuitively, a higher probability of detection reduces the output, unemployment, and welfare losses after a consolidation through tax hikes, since the incentives to reallocate to the informal sector are reduced. However, this effect is mostly seen in the short and medium run; in the long run, the results are similar for the different values of ρ.[10]

[8] Our estimates are broadly in line with those presented in recent studies that place the value of this elasticity in the (0.2,0.8) range. For a survey of the literature, see Saez et al. (2012).

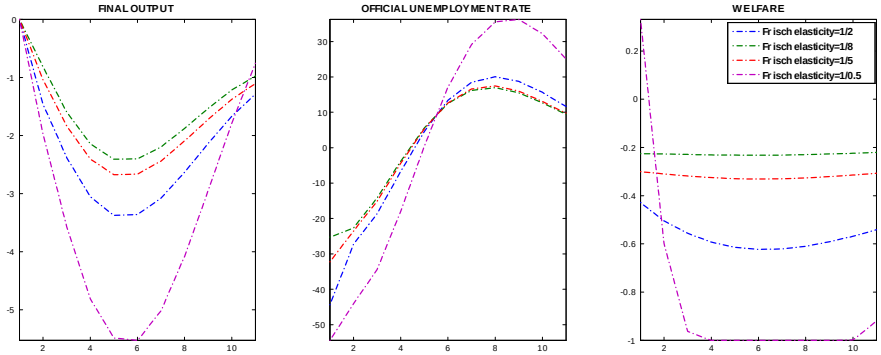
[9] Welfare comparisons are more difficult when we change η and φ because these parameters have a direct impact on the relative weight of consumption and leisure in the utility function.

[10] Since the auditing probability affects the reallocation of workers between sectors, it could also affect the consolidation through spending cuts. However, the results under the alternative values of ρ do not change substantially compared to the results of the baseline calibration.

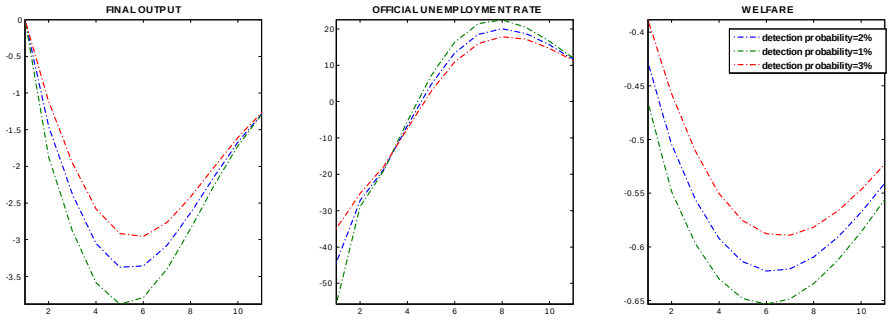
Figure 7: Sensitivity Analysis for Labor Tax Hikes in the Full Model



Intertemporal elasticity of substitution



Frisch labor supply elasticity



Detection probability

Table 2: Calibrated Values

Consolidation Volume - 2010 (% GDP)	7.8%
Consolidation Volume - 2015 (% GDP)	18.5%
Expenditure Share in Policy Mix	0.60

3 Policy Evaluation

We employ our DSGE model to analyze the effects of the recent consolidation packages implemented in Greece. Using the information in OECD (2012), we adjust the size of the consolidation to match the reduction in the deficit-to-GDP ratio implemented in 2010 and also replicate the announced consolidation volumes in the long run. Table 2 reports the consolidation in 2010 and the intended consolidation to be implemented by 2015. In order to replicate the actual consolidation packages, we allow both instruments to move simultaneously, again using OECD (2012) to fix the relative contribution of the two instruments.[11]

The simulation results are shown in Figure 8. Despite the substantial use of expenditure cuts, we see that tax evasion increases. With the use of tax hikes in the consolidation mix, the direct incentive to produce in the informal sector dominates the efficiency gains from producing in the formal sector, leading to a reallocation towards the informal sector. The model predicts sizeable and persistent output losses, unemployment increases in the long run and welfare losses following the consolidation packages.

4 Conclusions

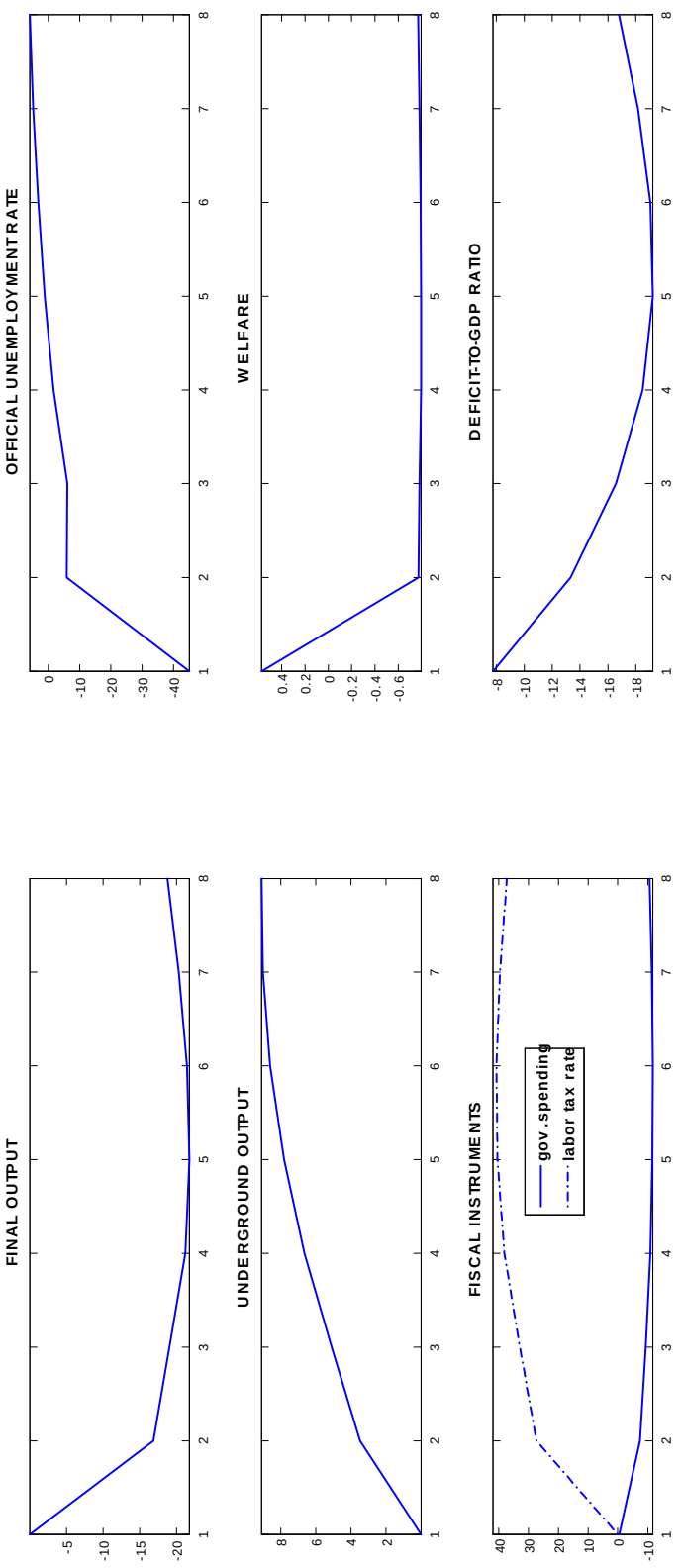
A New Keynesian DSGE model with involuntary unemployment and an informal sector demonstrates that the presence of the latter amplifies the contractionary effects of labor tax hikes, while mitigates the effects of expenditure cuts. It also shows that the instrument used to achieve fiscal consolidation affects the incentives of agents to produce in the informal sector. Spending cuts reduce the size of the informal economy, while tax hikes increase it.

We also analyze how current fiscal consolidation plans in Greece affect tax evasion, output, unemployment, and welfare. The model predicts increasing levels of tax evasion during the consolidation, and prolonged output and welfare losses. Greece suffers heavy losses due to the severity of the austerity package implemented. Furthermore, the welfare costs of these consolidations would have been smaller if tax evasion had been reduced. Hence, reforms aimed at fighting tax evasion should go hand-in-hand with austerity measures in order to mitigate the welfare costs of fiscal consolidations.

Our exercise is the first attempt to analyze the effects of fiscal consolidation in the presence of tax evasion. Since the model is stylized, it leaves out important aspects of reality that could affect our conclusions. For example, in our economy there is a representative household, and so we cannot assess the effects of tax evasion on income inequality. Also, we consider only cuts in government consumption expenditures and not in other items of the government budget. Furthermore, our

[11] See the table on p.138 of OECD (2012).

Figure 8: Simulation of Fiscal Consolidation Mix



model does not allow for evasion of consumption taxes, which is an important component of tax evasion in southern European countries. We leave these extensions for future research.

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5 Derivations

5.1 Household's maximization problem

We can write in full the Lagrangean for the representative household's maximization problem. Firstly, we can incorporate the composition of the household, as well as the definition of the total effective consumption bundle, directly into the utility function of the household. Then, we can plug the definition of the matches $m_t^j = \psi_t^{hj} u_t^j$ into the law of motion of employment in each sector, and also replace i_t in the budget constraint using the law of motion of private capital. Then we are left with 3 constraints, and the following Lagrangean:

$$\begin{aligned}\mathcal{L} = E_0 \quad & \sum_{t=0}^{\infty} \beta^t \Bigg\{ \frac{\left[\alpha_1 (c_t)^{\alpha_2} + (1-\alpha_1)(g_t)^{\alpha_2}\right]^{\frac{1-\eta}{\alpha_2}}}{1-\eta} + \Phi \frac{\left[1 - u_t - n_t^F - n_t^I\right]^{1-\varphi}}{1-\varphi} \\ & -\lambda_{ct}\Big[(1+\tau_t^c)c_t + k_{t+1} - (1-\delta)k_t + \frac{\omega}{2}\left(\frac{k_{t+1}}{k_t} - 1\right)^2 k_t + \frac{B_{t+1}\pi_{t+1}}{R_t} - r_t k_t \\ & -(1-\tau_t^n) w_t^F n_t^F - w_t^I n_t^I - \varpi(1-s_t)u_t - B_t - \Pi_t^p + T_t\Big] \\ & -\lambda_{n^F t}\Big[n_{t+1}^F - (1-\sigma^F) n_t^F - \psi_t^{hF}(1-s_t)u_t\Big] \\ & -\lambda_{n^I t}\Big[n_{t+1}^I - (1-\rho-\sigma^I) n_t^I - \psi_t^{hI} s_t u_t\Big] \Bigg\} \end{aligned} \tag{28}$$

The controls are c_t, k_{t+1}, B_{t+1}, n_{t+1}^F, n_{t+1}^I, u_t and s_t. The first order conditions are:
[wrt c_t]

$$c c_t^{(1-\eta-\alpha_2)} \alpha_1 c_t^{(\alpha_2-1)} - \lambda_{ct}(1+\tau_t^c) = 0 \tag{29}$$

[wrt k_{t+1}]

$$\lambda_{ct}\left[1 + \omega\left(\frac{k_{t+1}}{k_t} - 1\right)\right] - \beta E_t \lambda_{ct+1}\left[1 - \delta + r_{t+1} + \frac{\omega}{2}\left(\left(\frac{k_{t+2}}{k_{t+1}}\right)^2 - 1\right)\right] = 0 \tag{30}$$

[wrt B_{t+1}]

$$-\lambda_{ct}\frac{1}{R_t} + \beta E_t \lambda_{ct+1}\frac{1}{\pi_{t+1}} = 0 \tag{31}$$

[wrt n_{t+1}^F]

$$-\lambda_{n^F t} - \beta E_t\left[\Phi l_{t+1}^{-\varphi} - \lambda_{ct+1}(1-\tau_{t+1}^n) w_{t+1}^F - \lambda_{n^F t+1}(1-\sigma^F)\right] = 0 \tag{32}$$

[wrt n_{t+1}^I]

$$-\lambda_{n^I t} - \beta E_t[\Phi l_{t+1}^{-\varphi} - \lambda_{ct+1} w_{t+1}^I - \lambda_{n^I t+1}(1-\rho-\sigma^I)] = 0 \tag{33}$$

[wrt u_t]

$$-\Phi l_t^{-\varphi} + \lambda_{ct}\varpi + \lambda_{n^F t}\psi_t^{hF}(1-s_t) + \lambda_{n^I t}\psi_t^{hI} s_t = 0 \tag{34}$$

[wrt s_t]

$$-\lambda_{n^F t}\psi_t^{hF} u_t + \lambda_{n^I t}\psi_t^{hI} u_t - \lambda_{ct}\varpi = 0 \tag{35}$$

Equations (29)-(31) are the arbitrage conditions for the returns to consumption, private capital and bonds. Equations (32) and (33) relate the expected marginal value from being employed in the each sector to the wage, accounting for the income tax in the regular sector, the utility loss from the reduction in leisure, and the continuation value, which depends on the separation probability. Equation (34) states that the value of being unemployed (rather than enjoying leisure), $\lambda_{ct}\varpi$, should equal the marginal utility from leisure minus the expected marginal values of being employed in each sector, weighted by the respective job finding probabilities and shares of jobseekers. Equation (35) is an arbitrage condition according to which the choice of the share, s_t, is such that the expected marginal values of being employed, weighted by the job finding probabilities, are equal across the two sectors.

We can define the marginal value to the household of having an additional member employed in the two sectors, as follows:

$$\begin{aligned} V^h_{n^F t} &\equiv \frac{\partial \mathcal{L}}{\partial n_t^F} = \lambda_{ct} w_t^F (1-\tau_t^n) - \Phi l_t^{-\varphi} + (1-\sigma^F)\lambda_{n^F t} \\ &= \lambda_{ct} w_t^F (1-\tau_t^n) - \Phi l_t^{-\varphi} + (1-\sigma^F)\beta E_t(V^h_{n^F t+1}) \end{aligned} \tag{36}$$

$$\begin{aligned} V^h_{n^I t} &\equiv \frac{\partial \mathcal{L}}{\partial n_t^I} = \lambda_{ct} w_t^I - \Phi l_t^{-\varphi} + (1-\rho-\sigma^I)\lambda_{n^I t} \\ &= \lambda_{ct} w_t^I - \Phi l_t^{-\varphi} + (1-\rho-\sigma^I)\beta E_t(V^h_{n^I t+1}) \end{aligned} \tag{37}$$

where the second equalities come from equations (32) and (33) respectively.

5.2 Derivation of the resource constraint

Consider the household's budget constraint:

$$(1+\tau_t^c)c_t + i_t + \frac{B_{t+1}\pi_{t+1}}{R_t} \leq r_t k_t + (1-\tau_t^n) w_t^F n_t^F + w_t^I n_t^I + \varpi u_t^F + B_t + \Pi_t^p - T_t \tag{38}$$

Recall the government's budget constraint:

$$\frac{B_{t+1}\pi_{t+1}}{R_t} - B_t = DF_t$$

Plugging this into (38):

$$(1+\tau_t^c)c_t + i_t + DF_t \leq r_t k_t + (1-\tau_t^n) w_t^F n_t^F + w_t^I n_t^I + \varpi u_t^F + \Pi_t^p - T_t \tag{39}$$

Recall also the definition of the deficit:

$$DF_t = g_t + \varpi u_t^F - \left[(\tau_t^n + \tau_t^s) w_t^F n_t^F + \tau_t^c c_t + T_t\right] - \rho\gamma p_t^x x_t$$

Plugging this directly into equation (39):

$$(1+\tau_t^c)c_t + i_t + g_t + \varpi u_t^F - \left[\tau_t^c c_t + (\tau_t^n + \tau_t^s) w_t^F n_t^F + T_t\right] - \rho\gamma p_t^x x_t = \\ r_t k_t + (1-\tau_t^n) w_t^F n_t^F + w_t^I n_t^I + \varpi u_t^F + \Pi_t^p - T_t$$

Cancelling out the taxes and unemployment benefits, we have:

$$c_t + i_t + g_t - \rho\gamma p_t^x x_t = r_t k_t + (1+\tau_t^s) w_t^F n_t^F + w_t^I n_t^I + \Pi_t^p \tag{40}$$

Recall now that (i) the price of the final good is normalised to 1, (ii) the retail firms turn x_t units of the intermediate good into y_t units of the final good, and (iii) the differentiated retail goods are costlessly aggregated into the final consumption good. Then by definition, the profit from the retail firm can be written as:

$$\Pi_t^p = y_t - p_t^x x_t \tag{41}$$

Substituting this into equation (40), we obtain:

$$c_t + i_t + g_t = r_t k_t + (1+\tau_t^s) w_t^F n_t^F + w_t^I n_t^I + y_t - (1-\rho\gamma) p_t^x x_t \tag{42}$$

The price of the intermediate good, p_t^x, is determined by the zero-profit condition of the intermediate goods producing firm. That is, it satisfies:

$$\underbrace{(1-\rho\gamma)\, p_t^x x_t}_{\text{Revenue of intermediate firms}} - \underbrace{\left[(1+\tau_t^s) w_t^F n_t^F + w_t^I n_t^I + r_t k_t + \kappa^F v_t^F + \kappa^I v_t^I\right]}_{\text{Costs of intermediate firms}} = 0$$

Plugging this into equation (42):

$$\begin{aligned} c_t + i_t + g_t \;=\; & r_t k_t + (1\tau_t^s) w_t^F n_t^F \\ & + w_t^I n_t^I + y_t - \left[(1+\tau_t^s) w_t^F n_t^F + w_t^I n_t^I + r_t k_t + \kappa^F v_t^F + \kappa^I v_t^I\right] \end{aligned} \tag{43}$$

Cancelling terms we have:

$$c_t + i_t + g_t = y_t - \left(\kappa^F v_t^F + \kappa^I v_t^I\right) \tag{44}$$

Rearranging terms we get the final expression:

$$y_t = c_t + i_t + g_t + \kappa^F v_t^F + \kappa^I v_t^I$$

5.3 Derivation of the wages

For each sector $j = F, I$ the Nash bargaining problem is to maximize the weighted sum of log surpluses:

$$\max_{w_t^j} \left\{ (1-\vartheta^j) \ln V^h_{n^j t} + \vartheta^j \ln V^f_{n^j t} \right\}$$

where $V^h_{n^j t}$ and $V^f_{n^j t}$ are defined as:

$$V^h_{n^F t} = \lambda_{ct} w_t^F (1-\tau_t^n) - \Phi l_t^{-\varphi} + (1-\sigma^F)\lambda_{n^F t} \tag{45}$$

$$V^h_{n^I t} = \lambda_{ct} w_t^I - \Phi l_t^{-\varphi} + (1-\rho-\sigma^I)\lambda_{n^I t} \tag{46}$$

$$V^f_{n^F t} \equiv \frac{\partial Q}{\partial n_t^F} = (1-\rho\gamma)\, p_t^x (1-\alpha^F)\frac{x_t^F}{n_t^F} - (1+\tau_t^s) w_t^F + \frac{(1-\sigma^F)\kappa^F}{\psi_t^{fF}} \tag{47}$$

$$V^f_{n^I t} \equiv \frac{\partial Q}{\partial n_t^I} = (1-\rho\gamma)\, p_t^x (1-\alpha^I)\frac{x_t^I}{n_t^I} - w_t^I + \frac{(1-\rho-\sigma^I)\kappa^I}{\psi_t^{fI}} \tag{48}$$

The first order conditions of these optimization problems are:

$$\vartheta^F (1+\tau_t^s) V^h_{n^F t} = (1-\vartheta^F)\lambda_{ct}(1-\tau_t^n) V^f_{n^F t} \tag{49}$$

$$\vartheta^I V^h_{n^I t} = (1-\vartheta^I)\lambda_{ct} V^f_{n^I t} \tag{50}$$

Plugging the expressions for the value functions into these FOCs, we can rearrange to find expressions for w_t^F and w_t^I. Using (45), (47) and (49), we can solve for w_t^F, which yields:

$$w_t^F = \frac{(1-\vartheta^F)}{(1+\tau_t^s)} \left((1-\rho\gamma)\, p_t^x (1-\alpha^F)\frac{x_t^F}{n_t^F} + \frac{(1-\sigma^F)\kappa^F}{\psi_t^{fF}} \right) + \frac{\vartheta^F}{\lambda_{ct}(1-\tau_t^n)} \left(\Phi l_t^{-\varphi} - (1-\sigma^F)\lambda_{n^F t} \right) \tag{51}$$

Similarly using (46), (48) and (50), we can solve for w_t^I, which yields:

$$w_t^I = (1-\vartheta^I) \left((1-\rho\gamma)\, p_t^x (1-\alpha^I)\frac{x_t^I}{n_t^I} + \frac{(1-\rho-\sigma^I)\kappa^I}{\psi_t^{fI}} \right) + \frac{\vartheta^I}{\lambda_{ct}} \left(\Phi l_t^{-\varphi} - (1-\rho-\sigma^I)\lambda_{n^I t} \right) \tag{52}$$